Applicability of a Clustered Unit Commitment Model in Power System Modeling

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***Abstract*—Clustered unit commitment (CUC) formulations have been proposed to provide accurate and fast approximations to the unit commitment (UC) problem. In these formulations, identical or similar plants are grouped into clusters. This way, the binary commitment variables of all the plants within a cluster can be replaced by a single integer variable. This approach has recently been mainly used for tractably integrating flexibility constraints in generation expansion planning problems. However, a thorough general validation is still missing. In addition, these formulations do not provide commitment schedules on a plant-by-plant level and hence cannot be used directly for operating actual systems or markets. A first contribution of this paper is to show that errors can be introduced both due to the problem formulation and the grouping of non-identical units. A case study is presented in which these errors are quantified under different conditions. Overall, the error in approximating the total cost does not exceed 0.06%. A second contribution of this paper is the development of a hybrid approach which sequentially uses a CUC and a traditional UC model. This approach allows to reduce the computational cost of solving the UC problem while providing a guaranteed feasible and near optimal solution.**

***Index Terms*—Clustered unit commitment, power system modeling, mixed integer programming, unit commitment.**

NOMENCLATURE

Indices:

c, C	set of clusters.
i, I	set of power plants.
t, T	set of time steps.

Parameters:

Δ_t	time resolution [h]
D_t	demand at time step t [MW]
MC_i	slope of the linearized generation cost curve [Δ_t EUR/MWh]
MDT_i	minimum down time [Δ_t]
MUT_i	minimum up time [Δ_t]
N_c	number of units within cluster c [ϕ]
NC_i	intercept of the linearized generation cost curve [Δ_t EUR/h]
$\overline{P_i}$	maximum power output [MW]
$\underline{P_i}$	minimum power output [MW]
RD_i	maximum ramp-down rate [MW/Δ_t]
RU_i	maximum ramp-up rate [MW/Δ_t]
SD_i	maximum shut-down rate [MW]
SDC_i	shut-down cost [EUR]
SU_i	maximum start-up rate [MW]
SUC_i	start-up cost [EUR]

Variables:

$cost^{gen}_{i,t}$	generation cost [EUR]
$cost^{su}_{i,t}$	start-up cost [EUR]
$cost^{sd}_{i,t}$	shut-down cost [EUR]
$gen_{i,t}$	power generation [MW]
$n^{on}_{i,t}$	number of online units [ϕ]
$n^{sd}_{i,t}$	number of units shutting down [ϕ]
$n^{su}_{i,t}$	number of units starting up [ϕ]

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I. INTRODUCTION

TO limit the computational cost of solving unit commitment problems using mixed integer linear programming, several authors have aggregated power plants into a number of clusters [1]− [2]. This way, the binary commitment variables for all units within this cluster can be replaced by a single integer variable.

Gollmer et al. [1] have shown that by grouping thermal plants, the computation time can be reduced. However, they do not present their problem formulation. Sen and Kothari grouped together similar units. However, in their formulation, a binary instead of an integer commitment variable was used for every cluster, implying that either all plants within the cluster have to be online or all plants within the cluster have to be offline [3]. Improved formulations were presented in [4], [5] and [6]. In these formulations, a single or multiple integer variables are used to define the aggregate commitment state of all units within a cluster. In addition, the traditional UC formulation is adapted to account for the possible simultaneous ramping, starting up and shutting down of units within a single cluster.

Recently, the clustered unit commitment formulations have been mainly applied to integrate flexibility constraints in longer-term operational problems (e.g., asset valuation [5]) or generation expansion planning problems [7], [8]. For these problems, direct integration of binary unit commitment constraints is computationally infeasible. However, given the recent and expected further penetration of intermittent renewable energy sources (IRES), it becomes increasingly important to take into account the flexibility constraints in these models [5], [6], [8], [9]. According to [7], the CUC formulations speed up the computations by a factor in the order of 400-2000 depending on the approach used to group together different units.

This paper aims to address two gaps that can be found in the literature regarding clustered unit commitment formulations. A first gap relates to the validation of the problem formulation. In Langrene et al. [5], it is recognized that certain assumptions need to be made in formulating the dynamic constraints for a group of power plants. Nevertheless, the results produced using the clustered UC formulation have not been compared to those produced using a traditional binary unit commitment (BUC) model. In [7], a detailed comparison is made between a CUC and a traditional, binary unit commitment model. However, the focus in this work is on clustering of heterogenous units and on the impact of different strategies of clustering this heterogenous set of units on the approximation errors. In the literature it is assumed that, when using a single piecewise linear segment to model part-load efficiency losses, all approximation errors arise from clustering non-identical units, i.e., the formulation on its own does not induce errors. A second gap relates to the fact that the CUC formulations cannot be directly used for operating actual systems or markets as the output lacks information on a power plant basis [8].

This paper contributes the existing literature in two ways. First, we show that, aside from errors introduced by clustering non-identical units, errors inherent to the grouping of units into clusters can be introduced under some conditions. As such, we prove that clustering of identical units can lead to different results compared to a binary UC formulation. In a case study, we assess the magnitude of both errors inherent to the problem formulation and errors related to clustering non-identical units. Second, we present a novel hybrid methodology which combines the strengths of the CUC and BUC formulations. We show that this methodology can be used to provide near-optimal schedules on a plant-by-plant level at significantly reduced computation times.

In Section II, the traditional BUC formulation used in this paper is presented to compare against the CUC model. The mathematical formulation of the CUC model is presented in Section III. Section IV enlists two examples for which both formulations provide different results. A methodology and case study for assessing the model differences on a practical level are discussed in Sections V and VI respectively. Section VII presents the results and Section VIII concludes.

II. Binary Unit Commitment

The model displayed in this section is loosely based on the formulation presented by Van den Bergh et al. [10]. As the focus of this paper is on the representation of the load-following constraints of dispatchable power plants in a clustered UC formulation, details related to the representation of storage and variable renewable technologies, as well as system-based constraints such as reserve requirements and grid related constraints are not presented here. Similarly, tight and compact formulations (e.g, Morales-España et al. [11] and Ostrowski et al. [12]) are not considered in this paper as this would encumber the comparison between the BUC and CUC solutions in Section IV. The nomenclature typically used within UC formulations (e.g. u for the on/off state) is substituted by the symbols used in the clustered formulation to facilitate the comparison between both the traditional BUC and the CUC model.

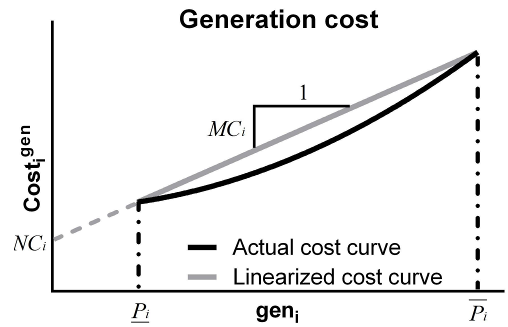


Fig. 1. Generation cost approximated by a single linear segment

A. Costs

The objective is to minimize the total operational cost. These operational costs comprise generation costs and costs incurred from start-ups and shut-downs:

$$min \sum_{i,t} (cost_{i,t}^{gen} + cost_{i,t}^{su} + cost_{i,t}^{sd}) \tag{1}$$

The generation cost is typically a non-linear function of the power output. This cost function can accurately be approximated by a set of linear piecewise segments. Here, only a single piecewise segment is considered for the whole operating range of the plant:

$$cost_{i,t}^{gen} = NC_i \cdot n_{i,t}^{on} + MC_i \cdot gen_{i,t} \quad \forall i,t \tag{2}$$

NC_i and MC_i are respectively the linearized fixed- and variable cost parameters of unit i (Fig. 1), $n_{i,t}^{on}$ is a binary variable which corresponds to the commitment status of plant i at time step t and $gen_{i,t}$ represents the output power of that plant.

The start-up and shut-down costs follow from:

$$cost_{i,t}^{su} = n_{i,t}^{su} \cdot SUC_i \quad \forall i,t \tag{3}$$
$$cost_{i,t}^{sd} = n_{i,t}^{sd} \cdot SDC_i \quad \forall i,t \tag{4}$$

$n_{i,t}^{su}$ ($n_{i,t}^{sd}$) is a binary variable which equals 1 if unit i starts up (shuts down) at time step t. SUC_i and SDC_i are parameters which contain the cost corresponding to these start-ups and shut-downs.

B. System Constraints

The market clearing constraint imposes demand-supply balance for each time step:

$$\sum_i gen_{i,t} = D_t \quad \forall t \tag{5}$$

The total generation of the individual plants must equal the demand D_t at every time step.

C. Technological constraints

Before dealing with the technological constraints, a logic relationship between the power plant states is needed:

$$n_{i,t+1}^{on} = n_{i,t}^{on} + n_{i,t}^{su} - n_{i,t}^{sd} \quad \forall i,t \tag{6}$$

Since the state of a plant can only be online or offline, the variables in Eq. 6 must be binary:

$$n^{on}_{i,t}, n^{su}_{i,t}, n^{sd}_{i,t} \in \{0,1\} \quad \forall i,t \tag{7}$$

Each plant has its minimum up and down time, this is represented in the following inequalities respectively:

$$n^{on}_{i,t} \geq \sum_{z=1:MUT_i} n^{su}_{i,t-z} \quad \forall i,t \tag{8}$$

$$1 - n^{on}_{i,t} \geq \sum_{z=1:MDT_i} n^{sd}_{k,t-z} \quad \forall i,t \tag{9}$$

Parameters MUT_i and MDT_i are the minimum up-time and minimum down-time of plant i respectively.

To avoid excessive thermal stresses, power plants can adjust their power output at a limited rate, the so-called ramping limits. The following constraints ensure that the unit operates within these ramping limits while also accounting for start-up and shut-down events:

$$\begin{aligned} gen_{i,t+1} - gen_{i,t} \leq & RU_i \cdot n^{on}_{i,t+1} \\ & + (SU_i - RU_i) \cdot n^{su}_{i,t} \quad \forall i,t \end{aligned} \tag{10}$$

$$\begin{aligned} gen_{i,t} - gen_{i,t+1} \leq & RD_i \cdot n^{on}_{i,t} \\ & + (SD_i - RD_i) \cdot n^{sd}_{i,t} \quad \forall i,t \end{aligned} \tag{11}$$

RU_i and RD_i are parameters which represent the maximum ramp-up and ramp-down between successive periods respectively. Parameter SU_i is the maximum amount of power output a plant can reach directly after a start-up and SD_i is the maximum generation level from where a plant is able to shut down.

Each plant is furthermore restricted to operate within a predefined power range delimited by a minimum and maximum operating point (see Fig. 1). The lower limit for the power output is:

$$gen_{i,t} \geq \underline{P_i} \cdot n^{on}_{i,t} \quad \forall i,t \tag{12}$$

Parameters $\underline{P_i}$ and $\overline{P_i}$ represent the minimum and maximum stable operating limits of unit i respectively. Power plants are assumed to start up to (and shut down from) a range starting from the minimum operating point. Therefore, the upper limit also takes into account these unit start-up and shut-down capabilities:

$$gen_{i,t} \leq \overline{P_i} \cdot n^{on}_{i,t} - (\overline{P_i} - SU_i) \cdot n^{su}_{i,t-1} \quad \forall i,t \tag{13}$$

$$gen_{i,t} \leq \overline{P_i} \cdot n^{on}_{i,t} - (\overline{P_i} - SD_i) \cdot n^{sd}_{i,t} \quad \forall i,t \tag{14}$$

Thus, a plant is able to start up to an output power in the range $[\underline{P_i}, SU_i]$ and is able to shut down from a generation level in the range $[\underline{P_i}, SD_i]$.

III. CLUSTERED UNIT COMMITMENT

Clustered UC formulations have been proposed to reduce the computational cost. This is done by clustering similar and/or identical units. For instance, Palmintier and Webster [7] consider clustering individual units by location and type (e.g., combined cycle gas turbine, natural gas combined cycle gas turbine, coal steam turbine, etc.), but also provide and evaluate different approaches for clustering a heterogeneous set of units into clusters. Accordingly, a single integer variable can be used to represent the number of online units within each cluster in every time step. As such, the number of variables and the state space can be strongly reduced compared to traditional BUC formulations which do consider individual plants by binary variables. The concept of clustering similar or identical units is illustrated in Fig. 2. Since this example contains a single cluster consisting of three units, the number of potential states in the binary formulation equals $2^3 = 8$ whereas the clustered formulation only contains 4 potential states. This benefit is vastly magnified by an increasing number of units within the cluster. E.g., a cluster comprising 30 individual units only contains 31 possible states in the CUC model, whereas the number of potential states in the binary formulation equals $2^{30} \approx 1.1 \cdot 10^9$. Hence, clustering eases the search through the extensive combinatorial commitment state space by eliminating a large number of identical or very similar commitment decisions. Additionally, clustering also reduces the number of continuous variables and constraints since they now only apply to a small number of clusters rather than the full set of separate generators [13].

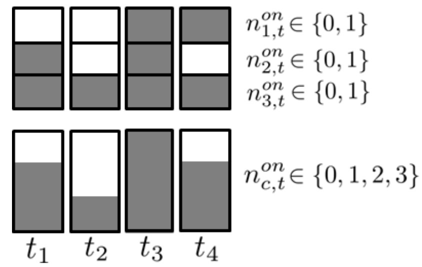


Fig. 2. Illustration of clustering three units in order to reduce the number of variables and the state space.

Clustering methods can be divided in several classes and there are different approaches to aggregation. For example, clustering can only be considered for identical units, but may also be implemented for similar, non-identical generators. This is a trade-off between accuracy of the solution and gain in calculation time. Fewer, but larger groups of similar units will secure a larger gain in calculation time, but the optimal solution will be affected by the fact that all units are assumed identical within every cluster. The cluster characteristics are defined as the mean parameters of the units within a cluster, thereby losing plant-level information.

Whenever internal congestion occurs frequently or security constraints strongly impact the solution, it might be important to consider a detailed grid representation. In these cases, clustering can only occur within nodes and the number of identical or similar plants per node might be very limited. As such, the computational advantage of a CUC model decreases when using models with a detailed network representation.

To the best of our knowledge, the two most advanced CUC approaches were developed by Palmintier and Webster [7] and Poncelet et al. [14]. The formulation presented here is based on the model by Palmintier and Webster as it closely resembles the BUC model. The original formulation is slightly adapted since it did not consider start-up or shut-down ranges in the generation limits (Eq. 13 - 14). The variables and parameters as defined in the nomenclature are valid here as well. The interpretation remains unchanged when replacing the unit index i by the cluster index c.

Mathematically, the CUC formulation necessitates few changes compared to the BUC model. The individual unit index i will be substituted by the cluster index c and additionally, the commitment variables no longer are binary variables, but can take on integer values:

$$n^{on}_{c,t}, n^{su}_{c,t}, n^{sd}_{c,t} \in \{0, 1, ..., N_c\} \quad \forall c, t \tag{15}$$

The amount of online units, start-ups and shut-downs within a cluster c is limited to the number of power plants N_c within that cluster.

Beyond the substitutions above, no further changes are required for the objective function (Eq. 1), generation cost (Eq. 2), start-up cost (Eq. 3), shut-down cost (Eq. 4), system balance (Eq. 5), logic condition (Eq. 6), minimum up time (Eq. 8) and the generation limits (Eq. 12 - 14).

The left hand side of the minimum down time limitation (Eq. 9) is limited to a binary value. In the CUC formulation, this number can take on values greater than one and thus, the constraint must be adapted:

$$N_c - n^{on}_{c,t} \geq \sum_{z=1:MDT_c} n^{sd}_{c,t-z} \quad \forall c, t \tag{16}$$

The ramping limits require the most extensive change with respect to the BUC ramping constraints (Eq. 10 - 11) since the power output of a cluster can change due to start-ups ($n^{su}_{c,t}$), shut-downs ($n^{sd}_{c,t}$) and the ramping of units that remain online. Moreover, the ramp rate limits of the latter are scaled by the number of online units within the cluster:

$$\begin{aligned} gen_{c,t+1} - gen_{c,t} \leq & (n^{on}_{c,t} - n^{sd}_{c,t}) \cdot RU_c - \underline{P_c} \cdot n^{sd}_{c,t} \\ & + SU_c \cdot n^{su}_{c,t} \quad \forall c, t \end{aligned} \tag{17}$$

$$\begin{aligned} gen_{c,t} - gen_{c,t+1} \leq & (n^{on}_{c,t} - n^{sd}_{c,t}) \cdot RD_c - \underline{P_c} \cdot n^{su}_{c,t} \\ & + SD_c \cdot n^{sd}_{c,t} \quad \forall c, t \end{aligned} \tag{18}$$

$(n^{on}_{c,t} - n^{sd}_{c,t})$ represents the number of units that stay online between periods t and t+1. These are the only units which can provide ramping capabilities.

IV. Deviations between both formulations

This section illustrates that in some situations, there will be differences between the results provided by a CUC and a BUC model, even if only identical plants are gathered into clusters. As such, we provide counterexamples for the prevailing idea in the state-of-the-art clustered unit commitment literature that CUC formulations provide identical solutions as BUC formulations whenever only identical units are clustered and part-load efficiency losses are modeled using a single piecewise linear segment. It is noteworthy to mention that all errors inherent to the CUC formulation discussed here arise from limiting start-up and shut-down ranges. No discrepancies were found when considering clusters not confined by these constraints. The implication of this result is that, in the case of limiting start-up and shut-down ranges, CUC formulations cannot guarantee optimal or feasible generation schedules even if only identical units are clustered. The dependence of the CUC model's accuracy on the flexibility of the considered portfolio will be assessed in Section VII.

The deviations will be illustrated using two simplified examples. In both examples, two identical plants are considered which need to satisfy a certain load. The two identical plants are grouped into a single cluster. The solutions to this problem provided by the BUC and the CUC formulations are compared. Numerous other examples can be identified. The illustrations in this section merely serve to illustrate the existence of deviations between both formulations from a theoretical perspective. For simplicity, these illustrations make use of a very small number of rather inflexible units and fairly large step changes in load, We realize that these situations do not reflect real power systems. In the next sections, we will focus on assessing whether and to what extent the differences between both formulations will lead to deviations in the results in realistic power systems.

Finally, load curtailment has been implemented in order to avoid infeasible solutions. This is severely penalized and thus, both models will only employ this option if absolutely unavoidable.

A. Illustration 1: overestimation of the shut-down capabilities

Consider a cluster comprising two identical units with the properties of Table I, and a demand profile as specified in Table II. Solving the illustration with both the BUC and the CUC formulation results in the solutions illustrated in Fig. 3. One may directly notice that the CUC formulation is able to solve this problem without the need of load curtailment, while the binary UC formulation can only obtain a feasible solution by shedding 50 MW during the second time step.

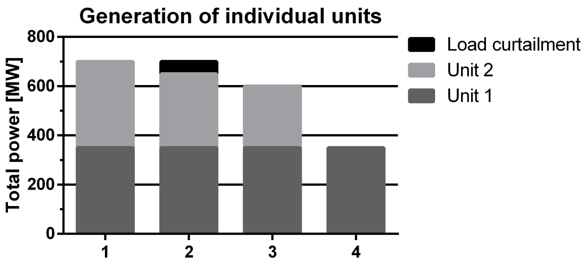
Fig. 3a represents the true, binary unit commitment solution. Both units are online during the first three periods and the second power plant shuts down between time steps 3 and 4. If the demand at time step 2 would be served, both power plants must be generating at their maximum operating points (350 MW). Moreover, in order for the second unit to be able to shut down between time steps 3 and 4, it has to be within its shut-down range (below 250 MW) at time step 3. These two situations cannot occur simultaneously because of the limited ramping rate of 50 MW/period. That is, if a unit is to shut

TABLE I
PROPERTIES OF THE INDIVIDUAL POWER PLANTS IN THE ILLUSTRATION 'OVERESTIMATION OF THE SHUT-DOWN CAPABILITIES'.

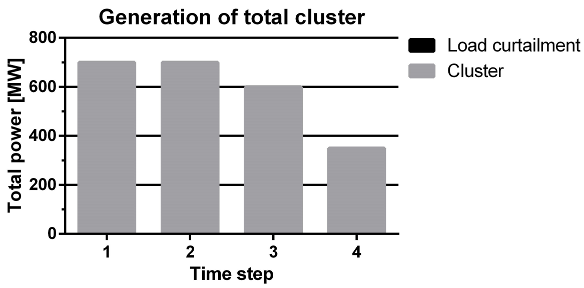
$\underline{P_i}$	$\overline{P_i}$	RU_i/RD_i	SU_i/SD_i	MUT_i/MDT_i
200 MW	350 MW	50 MW/period	250 MW	1 period

TABLE II
DEMAND REQUIREMENTS FOR THE CLUSTER IN THE ILLUSTRATION 'OVERESTIMATION OF THE SHUT-DOWN CAPABILITIES'.

Time step	1	2	3	4
Demand	700 MW	700 MW	600 MW	350 MW



(a)



(b)

Fig. 3. (a) Generation plots of the BUC solution for the illustration 'overestimation of the shut-down capabilities (Tables I and II). (b) As for (a) but for the CUC solution.

down between time steps 3 and 4, the maximum amount it may be producing at time step 2 is 300 MW, explaining the need for load curtailment.

Fig. 3b represents the clustered unit commitment solution. The commitment state cannot be inferred from the figure but is identical to the one in the BUC solution: two units are online during the first three periods and only one unit remains online thereafter. Since the individual unit data is unavailable, the constraints have to restrict the cluster's total generation. Between time step 2 and 3, the cluster's output ramps down with 100 MW, which is feasible since both units have a ramping capability of 50 MW. Between period 3 and 4, the cluster's output is further reduced by 300 MW, thereby providing a solution which supposedly does not require load curtailment. This reduction of output of 300 MW is achieved by ramping down 50 MW with the unit remaining online, while the other unit shuts down from an operating level of 250 MW. This solution does not violate any of the CUC constraints. However, due to the ramping constraint between time step 2 and 3, every plant had to be operating at 300 MW during time step 3 in order to serve the load. This is above the minimum shut down range, and hence the proposed schedule is infeasible.

Equivalently to the shut-down range, start-up ranges can be violated in the CUC model. This could be observed if one would reverse the demand profile (Table II).

These illustrations exemplify the exact issue, namely that the clustered UC formulation lacks information on the generation level of individual units. The CUC formulation only looks at the instantaneous generation of the entire cluster. However, as this example illustrates, the flexibility that can be provided by a set of plants does not only depend on the aggregate generation level of the entire cluster, but also on how this generation is distributed among the individual plants within the cluster. This distribution of the generation among plants within a cluster might not be chosen freely due to past or future events (e.g., the shut-down between time step 3 and 4, accompanied by limiting ramping capabilities, enforces one unit to be operating at 300 MW in time step 2).

It is noteworthy to mention that it is physically possible for units to disconnect from the grid at any load point. Although this type of shut-down does incur additional costs, it would avoid the load curtailment shown in Figure 3a. In the CUC formulation, these additional costs cannot fully be represented as the model would not acknowledge that units are shutting down from above their limits. In this context, the CUC model is less likely to generate infeasible solutions compared to the BUC formulation, but more likely to generate sub-optimal solutions as a proportion of the incurred costs are not being considered.

B. Illustration 2: violation of the maximum generation limits

The example in this subsection will illustrate that imposing minimum up time constraints might result in an overestimation of the maximum generation limits in the CUC formulation. Consider again a single cluster consisting out of 2 plants, but now with the properties of Table III and a demand profile as specified in Table IV.

The schedules provided by the BUC and CUC problem are displayed in Fig. 4. It can again be noticed that the CUC solution is able to follow the demand profile perfectly while the BUC solution requires load curtailment in order to obtain a feasible solution. This can be explained as follows:

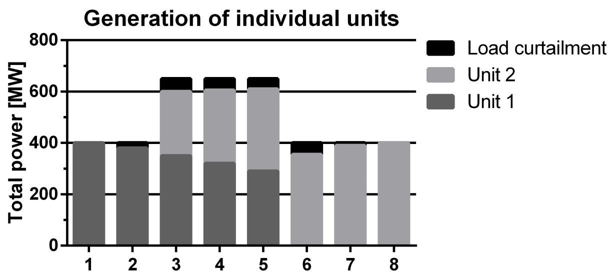
TABLE III
PROPERTIES OF THE INDIVIDUAL POWER PLANTS IN THE ILLUSTRATION 'VIOLATION OF THE MAXIMUM GENERATION LIMITS'.

$\underline{P_i}$	$\overline{P_i}$	RU_i	RD_i	SU_i	SD_i	MUT_i	MDT_i
200 MW	400 MW	35 MW/period	30 MW/period	250 MW	290 MW	4 periods	4 periods

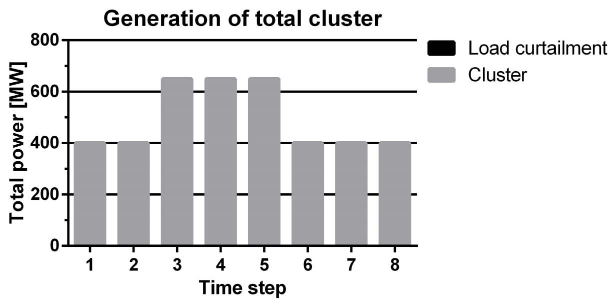
The valid, binary unit commitment solution is presented in Fig. 4a. During the third time step, an abrupt increase in demand calls for the start-up of the second power plant. Three periods later, the sudden cutback in demand requires the shut-down of one of the two online units. The start-up and the shut-down cannot be performed by the same unit due to the minimum up time of 4 periods. This, accompanied by limited start-up, shut-down and ramping capabilities, results in load curtailment over multiple periods.

TABLE IV
DEMAND REQUIREMENTS OF THE CLUSTER IN THE ILLUSTRATION 'VIOLATION OF THE MAXIMUM GENERATION LIMITS'.

Time step	1	2	3	4	5	6	7	8
Demand [MW]	400	400	650	650	650	400	400	400



(a)



(b)

Fig. 4. (a) Generation plots of the BUC solution for the illustration 'violation of the maximum generation limits' (Tables III and IV). (b) As for (a) but for the CUC solution.

The clustered unit commitment solution is presented in Fig. 4b. The commitment state cannot be inferred from the figure but is identical to the one in the BUC solution as the model observes the need for a start-up between time steps 2 and 3, and a shut-down between time steps 5 and 6. According to the minimum up time constraints (Eq. 8), one plant is allowed to shut down. However, due to the lack of distinctive unit information, the model does not recognize that it is the plant that was already online in time step 2 which needs to shut down later on. As a result, there is no need for this plant to start ramping down to make sure it is in its shut-down range by the time it needs to shut down. Even more so, apart from a single start-up and a single shut-down, the CUC formulation does not perceive any need for units to ramp (and possible associated ramping costs).

The presented examples illustrate that the CUC formulation does not keep track of the generation level of individual plants and, as such might relax technical constraints. Deviations between both formulations only have been found for clusters comprising units with limiting start-up and shut-down ranges. The implications are that, when modeling inflexible power plant systems, the solution provided by a clustered unit commitment model might be (i) infeasible and (ii) sub-optimal, even if only identical units are clustered.

V. Methodology

A. Metrics for comparison

Two different metrics for comparison are taken into account in order to provide results relevant to a range of applications.

The total cost is the objective function and includes all operational costs and curtailment penalties. As a metric for comparison, the relative cost error is used:

$$\Delta cost_{xy}^{tot} = \frac{cost_x^{tot} - cost_y^{tot}}{cost_{base}^{tot}} \tag{19}$$

Where x and y indicate the associated simulations which are being compared. The baseline will always be the solution of the BUC formulation for the considered portfolio.

The second metric is the average error in the projected fuel shares. As one of the main applications of the CUC model currently are generation expansion problems, it is useful to determine whether the clustered solution accurately represents the optimal generation mix.

$$\Delta E_{xy} = mean_{f \in F} |E_x^f - E_y^f| \tag{20}$$

The fuel share of plants consuming fuel of type f is defined as follows:

$$E^f = \frac{\sum_{t}^{N_t} \sum_{i \in f}^{N_i} gen_{i,t} \cdot \Delta_t}{\sum_{t}^{N_t} \sum_{i}^{N_i} gen_{i,t} \cdot \Delta_t} \tag{21}$$

The equation also is valid for the CUC formulation after substituting the unit index i by the cluster index c.

Finally, the computational effort of the different models is compared by keeping track of the calculation time.

B. Assessing the model differences

In the remainder of this work, we aim to assess what the impact is of both the errors inherent to the CUC formulation (formulation error) and the errors due to clustering non-identical units (clustering error) for a realistic case. In order to separate both sources of error, a fictional power-plant portfolio is created by changing the characteristics of the individual units to the characteristics of the cluster they belong to. By solving (i) the BUC problem with the original portfolio, (ii) the BUC problem with the fictional power-plant portfolio and (iii) the CUC problem, both sources of errors can be evaluated. First, comparing the solutions of the BUC model of the original and the fictional portfolio allows to assess the clustering error. Second, comparing the results of the BUC for the fictional portfolio and the CUC model, the formulation errors can be determined. Finally, the total error follows from comparing the BUC solution of the actual portfolio to the solution of the CUC model. The methodology is schematically represented in Fig. 5.

For incorporating a CUC model in generation expansion planning models, it is mainly important that performance measures, such as the operational cost and the energy mix are approximated with high accuracy. Therefore, the appropriate metrics will be evaluated for both the formulation and clustering error.

C. Proposal for a hybrid method

As discussed in Section IV, the feasibility of the commitment schedule as provided by the CUC model cannot be guaranteed when modeling inflexible power plant portfolios.

Additionally, CUC solutions do not contain the schedule of individual units since the variables are aggregated on a cluster level. The CUC model does consequently not generate applicable UC schedules that can be used within daily operations. This subsection proposes a hybrid approach which guarantees a final feasible solution on the unit level. The first step of this hybrid approach is to run the CUC model. In a second step, the solution of the CUC model is used to define additional constraints which are appended to a traditional BUC model. These constraints restrict the number of online units corresponding to a cluster to equal the results of the CUC model. In the BUC model, this can be mathematically implemented as:

$$\sum_{i \in c} n_{i,t}^{on} = n_{c,t}^{on} \quad \forall c, t \tag{22}$$

Here, the number of online units within a cluster $n_{c,t}^{on}$ is a parameter obtained by the preceding CUC model. An exception is made for fast-starting units for which no additional constraints are imposed. In the third and final step, the resulting BUC model is run. The BUC model is still able to determine which of the units within a cluster are chosen to be online. In other words, the CUC model determines the commitment state on a cluster level after which the BUC model refines this solution to obtain a feasible schedule on unit level. The additional constraints imposed in the BUC model strongly limit the solution space and thereby lower the calculation time. It must be noted however, that this hybrid method cannot guarantee to obtain the global optimal solution.

VI. Case Study

A. System description

To test the impact of clustering on a realistic power system, the Central Western European (CWE) electricity system was modeled using data from 2013 [15]. The considered system contains 806 power plants and 4 pumped storage units. The network model consists of 5 nodes (one for each country) and 5 lines. Additionally, the dataset contains load, wind, solar photovoltaic, conventional hydro, bio-energy and cogeneration time-series per node. To consider these system-extensions, the basic BUC and CUC models from Sections II and III are extended by load curtailment, renewable generation, renewable curtailment, transmission constraints and pumped storages (all implemented as in [10]). Load shedding and curtailment of renewables is possible at the very high cost of 10,000 EUR/MWh.

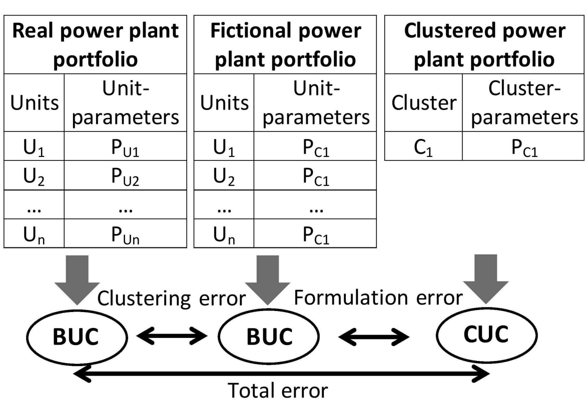


Fig. 5. Schematic representation of the methodology employed to separate the clustering error and the formulation error.

TABLE V
Overview of the range of technical cycling data [16] (NUC: nuclear power plants; SPPC: coal-fired steam power plants; SPP-L: lignite-fired steam power plants; SPP-G: gas-fired steam power plants; CCGT: combined-cycle gas turbines; OCGT: open-cycle gas turbines).

	Min. output [$\%\overline{P}$]	Ramping [$\%\overline{P}$/min]	Start-up/shut-down range [$\%\overline{P}$/switch]	Min. up time [h]	Min. down time [h]
NUC	40-50	0.25-5	50-100	0.25-24	24
SPP-C	25-40	0.66-4	40-100	0.25-10	3-10
SPP-L	40-60	0.66-4	60-100	0.25-10	3-10
SPP-G	40	0.83-6	40-100	0.25-6	1-6
CCGT	30-50	0.83-10	50-100	0.25-6	0.5-6
OCGT	20-50	0.83-25	50-100	0.25-1	0.25-1

Per simulation, three days were modeled simultaneously using an hourly resolution (72 time steps). All models are implemented in GAMS and solved using the CPLEX MILP solver. All runs were conducted on a Intel(R) Core(TM) i7-3770U CPU 3.40 GHz, 8 GB RAM with a target MILP tolerance of 0.005%.

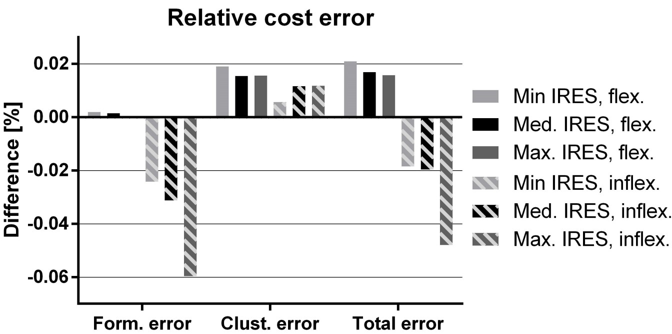
B. Parameter variations

In this case study, two parameters will be varied with the purpose of determining the accuracy of the CUC model under different conditions. First, the dependence of the CUC model's correctness on the flexibility of the considered power plant portfolio was already made clear in Section IV. In this paper, simulations are run for a flexible and an inflexible power plant portfolio. Both portfolios contain the same set of power plants, but with different cycling parameters. Van den Bergh and Delarue [16] provide an overview of the outer limits of these cycling parameters in the literature (Table V). In the inflexible portfolio, the power plants have stringent cycling parameters (see Table V, upper bound of minimum power output, lower bound of ramping gradients and upper bound of minimum up and down times). In the flexible portfolio, less constraining cycling parameters are used (see Table V, lower bound of minimum power output, upper bound of ramping gradients and lower bound of minimum up and down times).

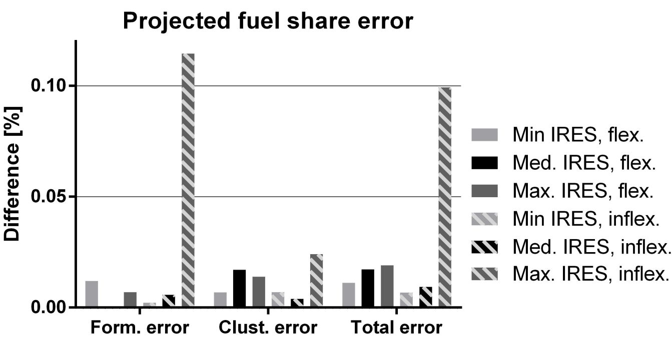
Second, the examples given in Section IV indicate that the deviations between the BUC formulation and the CUC model are triggered by fairly large step changes in residual load. Consequently, one might expect that the penetration of intermittent renewables would have an impact on the CUC model's accuracy. To this extent, simulations are run for three levels of IRES generation.

C. Clustering approach

Clustering occurs by type and age, e.g., all new combined cycle gas turbines (CCGT) are aggregated within the same



(a)



(b)

Fig. 6. (a) The formulation, clustering and total error for the relative cost whilst varying the level of intermittent renewable energy sources (IRES) and the flexibility of the portfolio. (b) As for (a) but for the projected fuel share.

group. The commissioning date here is important to allocate different efficiencies to clusters of the same unit type. Additionally, clustering of similar units is only allowed within the same node (country) to be able to account for cross-border transmission constraints. This results in a grouping of the 806 power plants into 99 clusters.

VII. Results

A. Model differences

Fig. 6a presents the relative cost error metric for the clustering error, the formulation error and the total error. A first observation that can be made is that the different errors for the presented case are very small.

A second observation concerns the impact of the portfolio's flexibility on the formulation error. For the inflexible portfolio, the formulation error exceeds the MIP gap (0.005%) and is negative in all cases. As discussed in Section IV, the CUC model relaxes certain constraints by not tracking the generation level of individual units. Hence, it can be expected that the CUC model is able to find a less expensive (but likely infeasible) solution. Also, the formulation error tends to increase for a higher share of IRES generation. A residual demand containing fairly large step changes thus triggers the deviations between the BUC formulation and the CUC model.

For the flexible portfolio, the formulation error never exceeds the MIP gap. In Section IV, deviations between both formulations only have been found for clusters comprising units with limiting start-up and shut-down ranges. The flexible portfolio used in this case study is not restricted by such constraints. This result suggests that both formulations are able to provide identical solutions when not considering stringent start-up and shut-down gradients.

A third observation that can be made is that the clustering error exceeds the MIP gap and is positive in all cases. By treating a heterogeneous set of units as identical, the costs tend to be overestimated. Consider for example a cluster consisting out of several units with slightly different generation costs. In the fictional portfolio, the cost-parameters will be uniform and equal to the average properties of these units. Hence, the model cannot make a distinction between these units and will be indifferent to the one which will be committed. In the real portfolio, the BUC model does make this distinction and will opt for the most inexpensive unit, resulting in a lower total cost. A similar reasoning can be made when considering the cycling parameters. Suppose that the demand profile requires a certain level of flexibility. It can be possible that a flexible unit within the real portfolio is just able to fulfill this demand. In the fictional portfolio however, the cycling parameters of this unit are based on the average values of the units within the cluster. Consequently, a single unit might not be able to fulfill the demand anymore and other solutions must be found, e.g., by starting up an additional unit or by using more flexible, but more expensive plants. It must be noted however, that in certain cases, it might be that the portfolio containing identical plants is able to find a lower cost solution, e.g., due to effects related to the capacity of individual units. Thus the positive sign of the clustering error cannot always be guaranteed. Nevertheless, for the presented case, the formulation error and the clustering error are different in sign, such that both errors tend to cancel each other to some extent. Finally, the clustering error remains fairly constant and does now show a clear dependence on IRES generation or on the portfolio's flexibility.

Over all simulated cases, the CUC model provides a gain in calculation time of factor $80-800$ when compared to the traditional UC formulation (Table VI). Note that these speed-up factors likely depend on the size of the case study as the calculation time of both approaches does not increase linearly with the number of variables. In addition, the speed-up factor might be higher when lower MIP gaps are used.

Finally, it is worth mentioning that none of the solutions employ load or renewable curtailment. The relative cost errors shown in Figure 6a only arise from differences in generation costs and costs incurred from start-ups and shut-downs.

In summary, the formulation cost error is negative and depends on the power plant portfolio's flexibility and on IRES production. The clustering cost error for the presented case is positive, but this cannot be generalized. Overall, the errors discussed here remain very small (<0.06%). Figure 6b presents the projected fuel shares error. Again, these errors remain very low (<0.12%) and thus, the clustered formulation is able to represent the optimal generation mix quite well. Hence, the use of CUC formulations in longer-term operational problems or generation expansion problems is justified.

TABLE VI
DIFFERENCES IN CALCULATION TIME BETWEEN THE SOLUTIONS PROVIDED BY THE TRADITIONAL UC MODEL AND THE CUC FORMULATION.

	Calculation time BUC [s]	Calculation time CUC [s]	Speed-up factor
Min. IRES, flex.	645,6	4,7	140
Med. IRES, flex.	370,3	4,2	87
Max. IRES, flex.	2116,1	4,0	533
Min. IRES, inflex.	2151,6	4,4	488
Med. IRES, inflex.	1546,7	8,3	185
Max. IRES, inflex.	2793,6	3,6	782

B. Performance of the hybrid model

For closer-term operations purposes, it is necessary to provide a commitment schedule on an individual unit level and guarantee the feasibility of this schedule. Consequently, a hybrid approach is proposed in which the commitment schedule of a CUC model is used to impose additional constraints in a subsequent BUC model. Table VII presents the different metrics of comparison between the result of this sequential approach and the original BUC solution. It may be seen that although the relative cost error often exceeds the MIP gap, the differences remain very small and thus, the sequential approach provides a near-optimal solution. The optimality difference is higher for the inflexible portfolio and increases for a greater share of IRES production. This corresponds to the inaccuracy of the CUC solution on which the commitment status is based.

The reason for this near-optimal solution is twofold. First, it was shown that the CUC solution accurately represents the operational cost and the energy-mix. The commitment status used to impose the additional constraints is therefore likely to be near-optimal. Second, the sequential BUC model is still able to determine which of the units within a cluster are chosen to be online. The model can also make economic dispatch decisions, and is able to employ other sources of flexibility such as storage and fast-starting units. As such, potential infeasibilities in the CUC commitment status are mitigated while curtailment options are limited to a minimum. It was already mentioned that none of the BUC and CUC solutions presented in this paper employ load or renewable curtailment. This is also true for all hybrid solutions but one. The exception is the first case shown in Table VI and curtails 0.035 MWh over the entire simulation. The sequential BUC model thus possesses a sufficient amount of flexibility to refine the CUC solution and to limit the additionally incurred costs.

In the final three columns of Table VII, the calculation time of the total hybrid approach is compared to the one of the traditional BUC model. It can be seen that time reductions of factor $1.1-40$ have been achieved depending on the considered parameter combination. Note that these speed-up factors again are likely to depend on the size of the case study as the calculation time of both approaches does not increase linearly with the number of variables. One might thus expect that the hybrid approach offers a higher speed-up potential for larger simulations. In addition, whenever internal congestion occurs frequently or security constraints strongly impact the solution, it might be important to consider a more detailed grid representation already in the CUC model. In this case, there might be less opportunities for clustering identical/similar units, and as such, the the hybrid methodology might be less advantageous.

Summarizing, in the presented case, the sequential CUC-BUC approach was able to generate a feasible near-optimal solution whilst reducing the solver run times significantly. In contrast to the CUC model, it generates individual unit schedules. Such an approach can be particularly useful whenever feasible solutions are imperative and the computation time is binding, e.g., in closer-term stochastic UC applications.

VIII. CONCLUSION

Clustered unit commitment (CUC) formulations have been developed to provide approximations for solving the UC problem. Recently, these formulations have been mainly applied for integrating flexibility constraints in long-term operational problems and generation expansion planning problems.

While these formulations have been applied frequently, in the literature the assumption is made that, under the premise of using a single piecewise linear segment to model part-load efficiency losses, a CUC formulation provides identical results to a traditional binary unit commitment formulation. In this paper, we demonstrate that this assumption only holds for a portfolio not restricted by start-up and shut-down limitations.

For clusters with stringent start-up and shut-down gradients, we show that errors can be introduced which are inherent to the problem formulation, i.e., even when only identical units are grouped into clusters, errors occur. These errors induced by the formulation are shown to originate from the fact that the CUC formulation does not keep track of the generation level of individual units. As the flexibility that can be provided by a group of power plants does not only depend on the aggregate generation level of all plants within the group, but also on how this generation level is distributed among different units, errors arise.

In a case study, both the errors inherent to the problem formulation and the errors induced by aggregating non-identical units have been quantified. It was shown that the CUC model tends to underestimate the true cost. Yet, this only is the case for the portfolio restricted by stringent start-up and shut-down gradients. The deviation between the solutions of the CUC model and the BUC formulation increase for a higher share of IRES generation and for a less flexible power plant portfolio. In the presented cases, the different types of errors remain very small. All relative cost errors and projected fuel share errors did not exceed 0.06% and 0.12% respectively. Provided a reliable clustering approach, the CUC formulation is thus able to accurately represent the total system cost and the optimal generation mix whilst reducing the calculation time by a factor 80-800. As such, CUC formulations are highly appropriate in longer-term operational problems or generation expansion problems.

Finally, a novel hybrid approach is presented in which a CUC and a BUC are run sequentially. The results of the CUC model are used to incorporate additional constraints in the

TABLE VII
DIFFERENCES BETWEEN THE SOLUTIONS PROVIDED BY THE TRADITIONAL UC MODEL AND THE HYBRID APPROACH.

	Relative cost error [%]	Projected fuel share error [%]	Calculation time BUC [s]	Calculation time hybrid [s]	Speed-up factor
Min. IRES, flex.	0.015	0.008	654.6	227.2	2.9
Med. IRES, flex.	0.002	0.003	370.3	116.6	3.2
Max. IRES, flex.	0.005	0.004	2116.2	52.9	40.0
Min. IRES, inflex.	0.016	0.005	2151.6	861.4	2.5
Med. IRES, inflex.	0.035	0.004	1546.7	1383.0	1.1
Max. IRES, inflex.	0.054	0.096	2793.6	684.1	4,1

BUC model in order to reduce the combinatorial state space of the BUC problem. As such, the computational cost of solving the UC problem can be reduced while providing a guaranteed feasible and near optimal solution on unit-level. The hybrid approach may be less advantageous when detailed network representations are important to consider. The effect of system size, clustering level and the impact of tight and compact constraints on the performance of this hybrid approach is left for future research.

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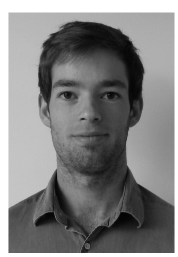

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